Strong Nash equilibria: complexity and algorithms

Nicola Gatti

November 5 2014

Non–cooperative game

agent 2

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agent 1	a_1	$2,2$	$0,3$
	a_2	$3,0$	$1,1$

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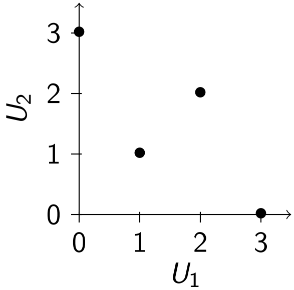

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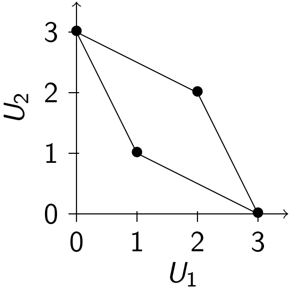
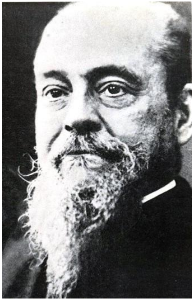


Solution concepts

- Game theory provides different solution concepts
- Each solution concept captures a different game situation
- Examples:
    - Can players communicate before playing and forming coalitions?
    - Can some player commit to a given strategy?
    - Can players correlate their strategies?

Solution concepts discussed in the talk

- **Pareto efficiency**: it captures the situation in which players cooperate



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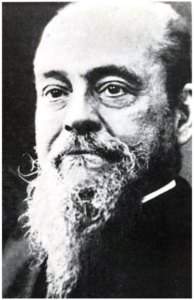
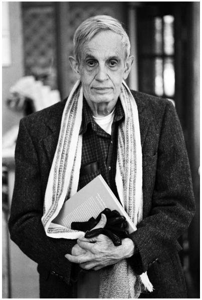
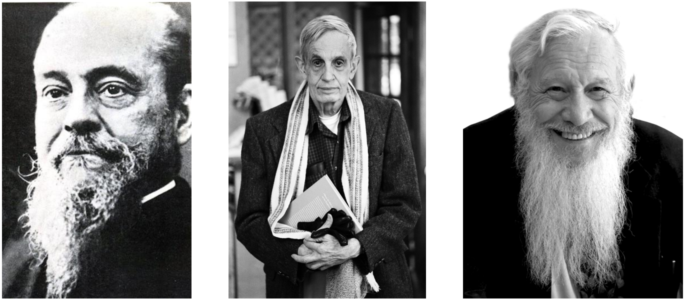
- **Pareto efficiency**: it captures the situation in which players cooperate
- **Nash equilibrium**: it captures the situation in which each player is selfish, doing its best given the strategy of the others





Solution concepts discussed in the talk

- **Pareto efficiency**: it captures the situation in which players cooperate
- **Nash equilibrium**: it captures the situation in which each player is selfish, doing its best given the strategy of the others
- **Strong Nash equilibrium**: it merges Nash equilibrium and Pareto efficiency for each possible coalition



Social efficiency: Pareto

Pareto Efficiency (PE)

A strategy profile is Pareto efficient if there is no other strategy profile in which:

- *all the players gain at least the same utility and*
- *at least one player gains strictly more.*

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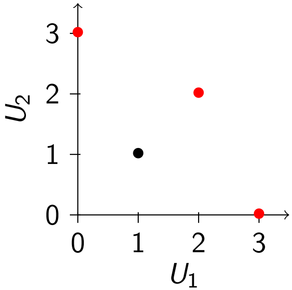

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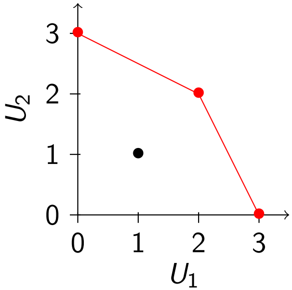

Pareto efficiency: properties

- Every game admits at least one Pareto efficient solution
- Finding a Pareto efficient solution and/or verifying whether a given solution is Pareto efficient could be not easy problems
- The behavior of rational players could be not socially efficient

Individual efficiency: Nash

Nash equilibrium (NE)

A strategy profile is a Nash equilibrium if no player can gain strictly more by unilateral deviations.

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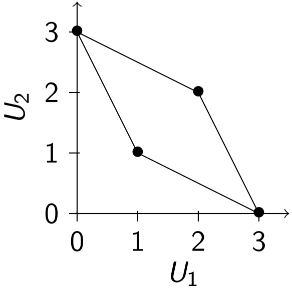

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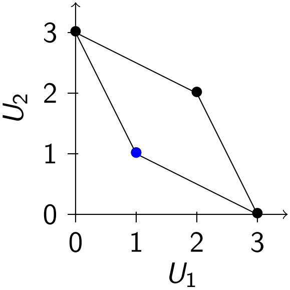

Nash equilibrium: properties

- Every finite game admits at least one Nash equilibrium in mixed strategies
- Finding a Nash equilibrium could be not an easy problem
- Nash equilibria may be not Pareto efficient

Selfishness with communication: Strong Nash

Strong Nash equilibrium (SNE)

A strategy profile is a Strong Nash equilibrium if:

- *it is a Nash equilibrium*
- *it is Pareto efficient for each possible sub coalition.*

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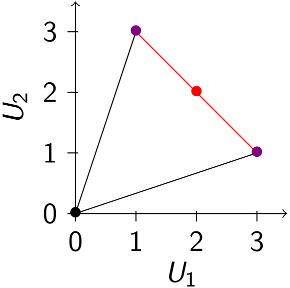

Strong Nash equilibrium: properties

- A Strong Nash equilibrium may not exist
- The verification problem involves the verification of two separate properties:
    - Pareto efficiency and
    - Nash equilibrium

  and therefore it could be not easy
- The finding problem could not be easy

Strong Nash equilibrium: in practice

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Strong Nash equilibrium: in practice

- Strong Nash would require that players can communicate and form coalitions
- In many scenarios, communication can be conducted by software agents operating over a network
- Internet is the main example
- In many scenarios, only some sub coalitions are possible (e.g. size–bounded sub coalitions)
- Graph–action games: games in which each agent is a node of a graph and its utility depends only on the neighbors' actions (*future research*)

Computation: motivation

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Computation: motivation

- In practice, we want to find solutions and therefore we need to answer to the question "What is the complexity of finding a solution?"
- If finding a solution is not achievable in reasonable time, then we need to relax a solution concept (e.g., ϵ–approximate NE)
- We resort to computational complexity tools

Computation: introduction

- Each computational problem is characterized by:
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  - **A question**

- We distinguish two main classes for the question:
  - **Decision**: the answer can be YES/NO
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- We would like to characterize problems in two main classes:
  - **Polynomial time**: problems admitting a polynomial–time (in the size of the input) algorithm
  - **Exponential time**: problems not admitting any polynomial–time (in the size of the input) algorithm

Computation: complexity classes (decision)

Polynomial ($\mathcal{P}$) There exists a polynomial–time algorithm

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Computation: complexity classes (decision)

Polynomial ($\mathcal{P}$)	There exists a polynomial–time algorithm
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$\mathcal{NP}$–hard	There exists a polynomial–time reduction from every $\mathcal{NP}$ problems

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$\mathcal{NP}$–hard	There exists a polynomial–time reduction from every $\mathcal{NP}$ problems
$\mathcal{NP}$–complete	The problem is in $\mathcal{NP}$ and it is $\mathcal{NP}$–hard

Computation: complexity classes (decision)

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Non–deterministic Polynomial ($\mathcal{NP}$) There exists a polynomial–time algorithm certifying that a solution given in input is a YES solution of the problem

$\mathcal{NP}$–hard There exists a polynomial–time reduction from every $\mathcal{NP}$ problems

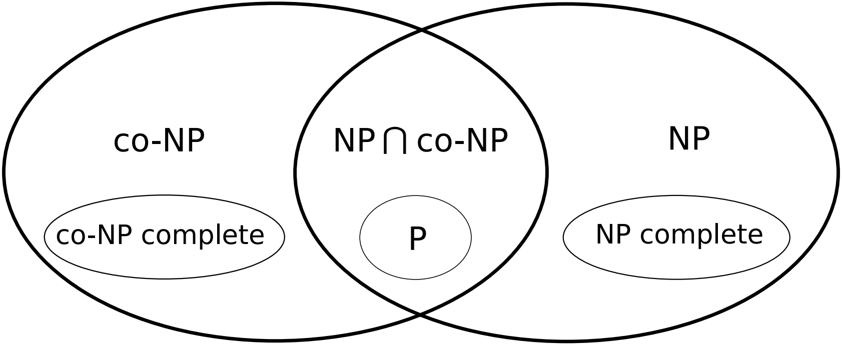
$\mathcal{NP}$–complete The problem is in $\mathcal{NP}$ and it is $\mathcal{NP}$–hard

Complement Non–deterministic Polynomial (co–$\mathcal{NP}$) There exists a polynomial–time algorithm certifying that a solution given in input is a NO solution of the problem

co–$\mathcal{NP}$–hard Analogous to $\mathcal{NP}$–hard

co–$\mathcal{NP}$–complete Analogous to co–$\mathcal{NP}$–hard

Computation: complexity classes (decision)



Decision problems and equilibria

PE existence	$\mathcal{P}$ class, given that there always exists a Pareto efficient solution

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Decision problems and equilibria

PE existence $\mathcal{P}$ class, given that there always exists a Pareto efficient solution

NE existence $\mathcal{P}$ class, given that there always exists a Nash equilibrium

SNE existence An SNE may not exist, addressing this task is not trivial; the problem has been proved $\mathcal{NP}$–complete

- $\mathcal{NP}$-hardness Proved by Conitzer and Sandholm (GEB, 2008) with 2–player symmetric games (from SAT)
- $\mathcal{NP}$ membership Proved by Gatti, Rocco, Sandholm (AAMAS, 2013)

NP–membership

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NE verification Can we certify in polynomial time that a given solution is an NE?

Yes, it is trivial

PE verification Can we certify in polynomial time that a given solution is an NE?

Yes, but the proof is not trivial

Nash equilibrium verification

		agent 2				
		a_6	a_7	a_8	a_9	a_10
agent 1	a_1	$2, 2$	$1, 3$	$3, 1$	$5, 2$	$3, 3$
	a_2	$3, 1$	$3, 0$	$0, 0$	$4, 0$	$4, 0$
	a_3	$1, 2$	$1, 1$	$2, 2$	$0, 1$	$0, 3$
	a_4	$3, 3$	$0, 2$	$0, 3$	$0, 4$	$1, 5$
	a_5	$4, 1$	$5, 0$	$1, 1$	$4, 0$	$3, 0$

Nash equilibrium verification

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		a_6	a_7	a_8	a_9	a_10
agent 1	a_1	2, 2	1, 3	3, 1	5, 2	3, 3
	a_2	3, 1	3, 0	0, 0	4, 0	4, 0
	a_3	1, 2	1, 1	2, 2	0, 1	0, 3
	a_4	3, 3	0, 2	0, 3	0, 4	1, 5
	a_5	4, 1	5, 0	1, 1	4, 0	3, 0

Nash equilibrium verification

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a_2	$3, 1$	$3, 0$	$0, 0$	$4, 0$	$4, 0$
a_3	$1, 2$	$1, 1$	$2, 2$	$0, 1$	$0, 3$
a_4	$3, 3$	$0, 2$	$0, 3$	$0, 4$	$1, 5$
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Nash equilibrium verification

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a_3	$1, 2$	$1, 1$	$2, 2$	$0, 1$	$0, 3$
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Pareto efficiency verification

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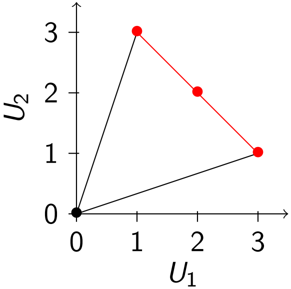

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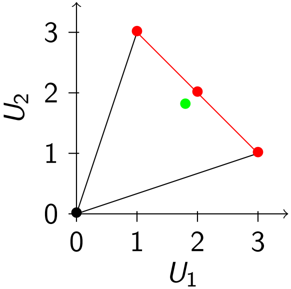

Pareto efficiency verification

Given in input $(\overline{x}_1, \overline{x}_2)$, we need to solve a feasibility problem

$$\mathbf{x}_1^t U_1 \mathbf{x}_2 - \overline{\mathbf{x}}_1^t U_1 \overline{\mathbf{x}}_2 > 0 \quad (1)$$

$$\mathbf{x}_1^t U_2 \mathbf{x}_2 - \overline{\mathbf{x}}_1^t U_2 \overline{\mathbf{x}}_2 > 0 \quad (2)$$

$$\mathbf{1}^t \cdot \mathbf{x}_1 = 1 \quad (3)$$

$$\mathbf{1}^t \cdot \mathbf{x}_2 = 1 \quad (4)$$

$$\mathbf{x}_1 > \mathbf{0} \quad (5)$$

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$$\mathbf{x}_1 > \mathbf{0} \tag{5}$$

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The problem (1)–(6) is:

- Non–linear
- Non–convex

Result

Theorem (PE verification is easy)

There is a polynomial–time algorithm verifying whether a given strategy profile is Pareto efficient when the number of players is fixed.

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There is a polynomial–time algorithm verifying whether a given strategy profile is Pareto efficient when the number of players is fixed.

Corollary

The problem of deciding whether a game admits an SNE is in $\mathcal{NP}$ *when the number of players is fixed.*

Result

Theorem (PE verification is easy)

There is a polynomial–time algorithm verifying whether a given strategy profile is Pareto efficient when the number of players is fixed.

Corollary

The problem of deciding whether a game admits an SNE is in $\mathcal{NP}$ *when the number of players is fixed.*

Corollary (PE finding is easy)

There is a polynomial–time algorithm returning a (generic) Pareto efficient solution when the number of players is fixed.

Proof sketch [PE verification is easy]

Step 1 We can formulate the problem as a minmax problem with

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Step 4 The optimal solution can be found by solving the minmax problem for $O(m^{n^2})$ supports

Summary The total complexity is $L^{O(1)} \cdot (n+1)^{O(n^2)} \cdot O(m^{n^2})$

Example

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Theory vs. practice (1)

Real algebraic geometry algorithms

- Mathematica
- Matlab
- Maple (SyNRAC)
- QEPCAD

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More than 3 hours for solving a single $3 \times 3 \times 3$ problem!

Theory vs. practice (2)

Global optimization solvers (tolerance = 10^{-14})

- SCIP
- BARON

Size (polymatrix games)	Average time	
	SCIP	BARON
2 players, 2 actions per player	< 1 s	< 1 s
3 players, 3 actions per player	~ 20 s	< 1 s
4 players, 4 actions per player	> 1 h	< 1 s
5 players, 5 actions per player	> 1 h	< 1 s
6 players, 6 actions per player	> 1 h	< 1 s
7 players, 7 actions per player	> 1 h	~ 2 s
8 players, 8 actions per player	> 1 h	~ 3 s
9 players, 9 actions per player	> 1 h	~ 4 s
10 players, 10 actions per player	> 1 h	~ 6 s

Theory vs. practice (3)

BARON — whole problem — (average time)					
Players	10 actions	20 actions	30 actions	40 actions	50 actions
2	0.04 s	0.15 s	0.45 s	1.13 s	1.31 s
3	0.09 s	0.76 s	1.73 s	2.02 s	2.74 s
4	0.34 s	2.22 s	2.90 s	3.85 s	5.72 s
5	1.02 s	3.17 s	3.88 s	6.35 s	10.40 s
6	1.85 s	3.86 s	6.03 s	10.13 s	18.41 s
7	3.62 s	5.10 s	8.34 s	16.03 s	30.74 s
8	4.34 s	6.13 s	11.19 s	24.27 s	48.33 s
9	4.96 s	8.27 s	15.84 s	35.26 s	72.43 s
10	5.65 s	9.27 s	20.99 s	49.19 s	106.62 s

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Step 1 We verify whether $\bar{\mathbf{x}}$ is a NE

Step 2 For each sub coalition

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Step 3 We check whether the strategy of the sub coalition is PE given the strategies of the other players

The number of sub coalitions are $2^n - 1$

SNE verification can be solved in polynomial time in the number of actions available to the players

SNE finding

Corollary

There exists a polynomial–time algorithm finding (if exists) a pure–strategy SNE.

Step 1 Scan all the pure strategy profile

Step 2 Check whether it is an SNE

Considerations

Question Why is the decision problem $\mathcal{NP}$–complete?

Considerations

Question	Why is the decision problem $\mathcal{NP}$–complete?
Answer	Because there can exist mixed–strategy SNEs with arbitrarily large supports.

Considerations

agent 2

		a_4	a_5	a_5
agent 1	a_1	$2, 2$	$0, 3$	$0, 3$
	a_2	$3, 0$	$1, 1$	$0, 3$
	a_3	$3, 0$	$1, 1$	$0, 3$

SNEs and mixed strategies

Question How many games admit mixed–strategy SNEs?

SNEs and mixed strategies

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SNEs and mixed strategies

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Answer The set is negligible.

Question What is the probability that a game randomly drawn with uniform probability from the set of all the games admits a mixed–strategy SNE?

Answer Zero.

SNEs and mixed strategies

Question	How many games admit mixed–strategy SNEs?
Answer	The set is negligible.
Question	What is the probability that a game randomly drawn with uniform probability from the set of all the games admits a mixed–strategy SNE?
Answer	Zero.

Therefore, with probability 1, a game either admits a pure–strategy SNE or does not admit any SNE

SNEs and perturbations

agent 2

		a_3	a_4
agent 1	a_1	$2, 0$	$0, 2$
	a_2	$0, 2$	$2, 0$

SNEs and perturbations

		agent 2	
		a_3	a_4
agent 1	a_1	$2, 0$	$0, 2$
	a_2	$0, 2$	$2, 0$

		agent 2	
		a_3	a_4
agent 1	a_1	$2+\epsilon_1, 0+\epsilon_2$	$0+\epsilon_5, 2+\epsilon_6$
	a_2	$0+\epsilon_3, 2+\epsilon_4$	$2+\epsilon_7, 0+\epsilon_8$

SNEs and perturbations

		agent 2	
		a_3	a_4
agent 1	a_1	$2, 0$	$0, 2$
	a_2	$0, 2$	$2, 0$

		agent 2	
		a_3	a_4
agent 1	a_1	$2+\epsilon_1, 0+\epsilon_2$	$0+\epsilon_5, 2+\epsilon_6$
	a_2	$0+\epsilon_3, 2+\epsilon_4$	$2+\epsilon_7, 0+\epsilon_8$

If ϵ_i are i.i.d. random variables in $[0, \overline{\epsilon}]$ even with small $\overline{\epsilon}$, the perturbed game admits a mixed–strategy SNE with zero probability.

Smoothed complexity

Definition (Smoothed $\mathcal{P}$)

A problem is in Smoothed $\mathcal{P}$ if there is an algorithm whose expected compute time is polynomial once the problem has been subject to perturbations.

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A problem is in Smoothed $\mathcal{P}$ if there is an algorithm whose expected compute time is polynomial once the problem has been subject to perturbations.

Theorem (SNE is in Smoothed $\mathcal{P}$)

The problem of deciding whether a game admits an SNE is in Smoothed $\mathcal{P}$ *when the number of players is fixed.*

Proof sketch

Algorithm

- Step 1 Scan all the pure strategy profiles and check whether they are SNEs
- Step 2 Check a zero–probability condition necessary for mixed–strategy SNEs
- Step 3 If yes, enumerate all the support profiles and check whether they admit SNEs

Considerations

- Engineering problems are characterized by errors in measurements and noisy
- Errors in measurements and noisy can be seen as perturbations
- In practice, engineering problems admit mixed–strategy SNEs with zero probability
- Only pure–strategy SNEs can be present

Conclusions

Pareto efficiency verification	$\mathcal{P}$
Pareto efficiency finding	$\mathcal{FP}$
Strong Nash equilibrium verification	$\mathcal{P}$
Strong Nash equilibrium decision	$\mathcal{NP}$–complete
Strong Nash equilibrium decision	Smoothed $\mathcal{P}$

Topics non–discussed in the talk

- Algorithms for finding SNEs
- Algorithms for enumerating the PE solutions

Future works

- Computational study of Strong Nash in special cases
    - Graph–action games
    - Speeding up pure–strategy SNE finding algorithms

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- Computational study of Strong Nash in special cases
    - Graph–action games
    - Speeding up pure–strategy SNE finding algorithms
- Computational study of other solution concepts
    - Strong Correlated equilibrium
    - Coalition–proof equilibrium