

# Marie Curie Workshop on Games, Logic, and Decision Theory

June 9, 2016

Sala di rappresentanza (ground floor)  
Dipartimento di Matematica *Federigo Enriques*  
Università degli Studi di Milano  
via Cesare Saldini 50  
Milano, Italy



Different game-theoretic models have common foundations based on utility theory, convex structures, logic, order theory, and other mathematical disciplines. This leads to a rich interaction amongst economists, mathematicians, and philosophers. The aim of the workshop is to gather researchers in game and decision theory in order to foster communication between experts working in different fields.

## Speakers

Alfredo Di Tillio	Università Bocconi
Mario Gilli	Università degli Studi di Milano-Bicocca
Michel Grabisch	Université Paris 1 Panthéon-Sorbonne
Roberto Lucchetti	Politecnico di Milano
Massimo Marinacci	Università Bocconi
Vincenzo Marra	Università degli Studi di Milano

## Program

9	–	9.15	Opening	Tomáš Kroupa
9.15	–	10.15	<i>On the vertices of the core</i>	Michel Grabisch
10.15	–	11.15	<i>Equilibria in a symmetric contest model</i>	Mario Gilli
			Coffe break	
11.45	–	12.45	<i>Strong Nash equilibria and mixed strategies</i>	Roberto Lucchetti
			Lunch	
14.45	–	15.45	<i>Remarks on game theory in a non-classical logical setting</i>	Vincenzo Marra
15.45	–	16.45	<i>Non Bayesian measurement</i>	Massimo Marinacci
			Coffe break	
17.15	–	18.15	<i>Conditional belief types</i>	Alfredo Di Tillio

Attendance is free and most welcome. The interested participants are kindly asked to send a message to the organizers.

## Organizers

Tomáš Kroupa	<a href="mailto:tomas.kroupa@unimi.it">tomas.kroupa@unimi.it</a>
Vincenzo Marra	<a href="mailto:vincenzo.marra@unimi.it">vincenzo.marra@unimi.it</a>

## Abstracts

Alfredo Di Tillio — *Conditional belief types*

We study type spaces where a player's type at a state is a conditional probability on the space. We axiomatize these spaces using conditional belief operators, examining three additional axioms of increasing strength. First, introspection, which requires the agent to be unconditionally certain of her beliefs. Second, echo, according to which the unconditional beliefs implied by the condition must be held given the condition. Third, determination, which says that the conditional beliefs are the unconditional beliefs that are conditionally certain. Echo implies that conditioning on an event is the same as conditioning on the event being certain, which formalizes the standard informal interpretation of conditional probability. The game-theoretic application of our model, discussed within an example, sheds light on a number of issues in the analysis of extensive form games. Type spaces are closely related to the sphere models of counterfactual conditionals and to models of hypothetical knowledge.

Mario Gilli (with Fausto Cavalli and Ahmad Naimzada) — *Equilibria in a symmetric contest model*

In this paper we propose a symmetric two player general contest model to analyze the characteristics of the set of equilibria as a function of possible outcomes and of structural characteristics of contest technology, given a general specification of the players' set of possible entries, of the agents' utility functions, and of the rules that presides over outcomes. The purpose of this paper is to study the relationship between equilibria and crucial structural parameters, in particular the marginal productivity of agents' efforts and the polarization between agents' goals, with and without spillover. The paper contains three main contributions. First, we show that without spillover the equilibrium efforts' intensity is uniquely connected to the ratio between marginal productivity of effort and polarization. Secondly, we are able to connect existence of multiple symmetric and asymmetric equilibria to the intensity of spillover effects into outcomes. Finally, we show that spillover in contest technology implies non existence of a pure strategy equilibrium

Michel Grabisch (with Peter Sudhölter) — *On the vertices of the core*

The core of a capacity or a game  $v$  on  $N$  is the set of additive games dominating  $v$  and coinciding on  $N$ . It is an important concept both in decision theory (set of compatible probability measures) and in cooperative game theory (set of coalitionally rational payoff vectors). When nonempty the core is a convex bounded and closed polyhedron. When the game is supermodular, it is well known that the vertices of the core are the so-called marginal vectors of the game. However, when the game is not supermodular, there is no general result giving the vertices of the core. Our aim is to investigate this problem in a more general framework, that is, when the game  $v$  is not defined on the entire Boolean lattice  $2^N$  but only on a distributive sublattice of it, generated by a partially ordered set on  $N$ , which can be interpreted as a hierarchy on  $N$ . We propose a way to produce vertices of the core based on particular orders on  $N$  inducing payoff vectors maximizing or minimizing the payoff given to the current player in  $N$  in the corresponding reduced game. We show however by a counterexample that this procedure cannot produce all vertices in general.

Roberto Lucchetti (with Eleonora Braggion, Nicola Gatti and Tuomas Saldhom) — *Strong Nash equilibria and mixed strategies*

We consider strong Nash equilibria, in mixed strategies, for finite games. First, we analyze the two-player setting. Our main result, in its simplest form, states that if a game has a strong Nash equilibrium with full support, then the game is strictly competitive. Our characterization enables us to design a strong Nash equilibrium finding algorithm with complexity in Smoothed- $\mathcal{P}$ . So, this problem that in general is computationally hard in the worst case is generically easy. Hence, although the worst case complexity of finding a strong Nash equilibrium is harder than

that of finding a Nash equilibrium, once small perturbations are applied, finding a strong Nash is easier than finding a Nash equilibrium. Next we switch to the setting with more than two players. We demonstrate that a strong Nash equilibrium can exist in which an outcome that is strictly Pareto dominated by a Nash equilibrium occurs with positive probability. Finally, we prove that the set of games having a strong Nash equilibrium where at least one player puts positive probability on at least two pure strategies is extremely small in any respect (in Baire sense, of null measure,  $\sigma$ -porous, sparse).

Massimo Marinacci — *Non Bayesian measurement*

We consider measurement problems from a non Bayesian viewpoint.

Vincenzo Marra — *Remarks on game theory in a non-classical logical setting*

A reading of the standard notions of game theory through the lenses of classical logic points towards possible generalisations in which the ground logic is taken to be non-classical. I try to show by example that such generalisations may hold potential interest in a number of ways, not least because of the non-trivial mathematical challenges they pose.