

## MATCHING MARKETS

- DEFINE A BIPARTITE GRAPH
- DEFINE A MATCH IN A BIPARTITE GRAPH
- DEFINE A PERFECT MATCH
- HOW CAN THE PROBLEM OF FINDING A PERFECT MATCH ~~BE~~ BE FORMULATED IN MATHEMATICAL PROGRAMMING FASHION?
- WHAT IS THE DIFFERENCE BETWEEN THE MATCHING PROBLEM AND THE ASSIGNMENT PROBLEM
- HOW CAN THE PROBLEM OF FINDING THE BEST ASSIGNMENT BE FORMULATED IN MATHEMATICAL PROGRAMMING FASHION?
- WHAT IS THE COMPLEXITY OF SOLVING THE ASSIGNMENT PROBLEM?
- DESCRIBING THE PUNLAWING OF THE HUNGARIAN ALGORITHM
- SOLVE THE ASSIGNMENT PROBLEM WITH MATRIX:

10	5	6
2	3	10
1	7	4

SOLUTION:

SINCE WE WANT TO MAXIMIZE, WE NEED TO CHANGE THE SIGN TO THE ENTRIES OF THE MATRIX

-10	-5	-6
-2	-3	-10
-1	-7	-4

THEN WE MAKE ALL THE ENTRIES POSITIVE BY ADDING  
~~THE~~ A CONSTANT (10)

0	5	4
8	7	0
9	3	6

SUBTRACT THE ROW MINIMUM FOR EACH ROW

0	5	4
8	7	0
6	0	3

SUBTRACT THE COLUMN MINIMUM  $\rightarrow$  NO EFFECT OTHER  
 THAT EACH COLUMN CONTAINS ZEROS

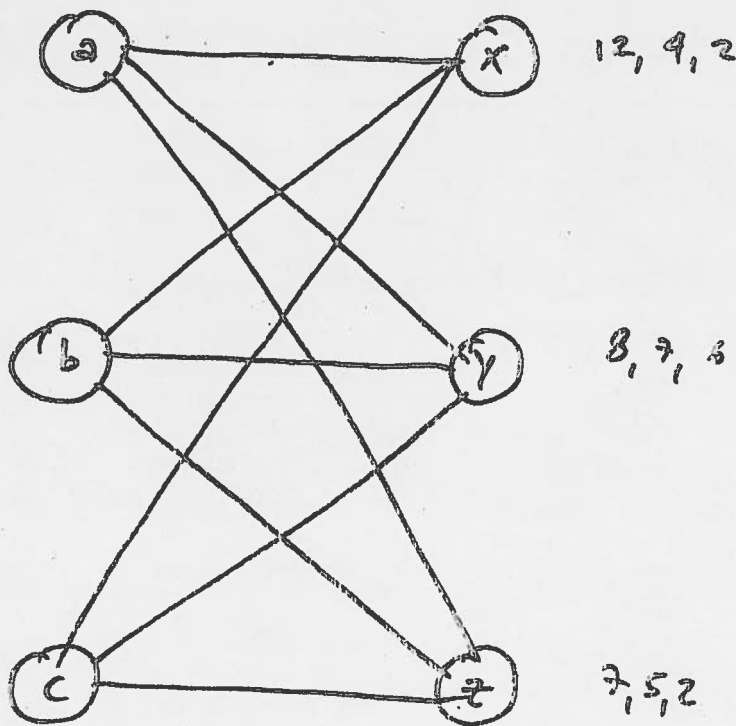
COVER ALL ZEROS WITH A MINIMUM NUMBER OF LINES


NO LINE CAN COVER MORE THAN ONE  
 ZERO

$\Rightarrow$  OPTIMAL ASSIGNMENT

10	5	6
2	3	10
1	7	4

- DESCRIBE MATCHING PROBLEMS BETWEEN BUYERS AND SELLERS
- DEFINE THE PREFERRED-SELLER GRAPH ON BUYERS AND SELLERS
- WHEN A SET OF PRICES IS MARKET-CLEARING?
- FIND THE MARKET-CLEARING PRICES USING THE VCG IN THE FOLLOWING PROBLEM



IN ORDER TO FIND  $(p_1, p_2, p_3)$  THAT ARE MARKET-CLEARING, IT IS NECESSARY TO APPLY THE VCG MECHANISM

- WE NEED TO FIND THE BEST ALLOCATION OF BUYERS TO SELLERS
- WE NEED TO COMPUTE THE VCG PAYMENTS

OPTIMAL ALLOCATION :

a b c  
x y z  $\rightarrow 21$

a b c  
x z y  $\rightarrow 23$

a b c  
y x z  $\rightarrow 14$

a b c  
y z x  $\rightarrow 15$

a b c  
z x y  $\rightarrow 17$

a b c  
z y x  $\rightarrow 16$

OPTIMAL ALLOCATION  $\rightarrow$

$(x, z, y)$   
↓ ↓ ↓  
 $(a, b, c)$

$$P_x = 14 - 11 = 3 \longrightarrow P_d$$

$$P_y = 19 - 18 = 1 \longrightarrow P_c$$

$$P_c = 0 \longrightarrow P_b$$