INFORMATION CASCADES AND INFLUENCE

MOTIVATION: NETWORK OF INTERACTIONS ARE FUNDAMENTAL TO SHAPING THE WAY IN WHICH IDEAS, INFORMATION AND INFLUENCE SPREADS THROUGH A POPULATION OCCURRING BETWEEN PAIRS OF PEOPLE WITH EXISTING RELATIONSHIPS

GOAL: UNDERSTANDING HOW THE STRUCTURE OF NETWORKS AND DYNAMICS ON NETWORKS AFFECT THE PROPAGATION OF IDEAS AND BEHAVIORS

EXAMPLES:

- THE ADOPTION OF NEW PRODUCTS OCCURS ALMOST ENTIRELY THROUGH "WORD-OF-MOUTH" EFFECTS
- SPREAD OR POLITICAL MEMES
- IDENTIFYING INFLUENCERS FOR THE PURPOSE OF VIRAL MARKETING WHERE SOME SMALL NUMBER OF PEOPLE ARE TARGETED TO BE EARLY ADOPTERS OF A NEW PRODUCT

MANY EMPIRICAL RESULTS OVER WHICH MODELS ARE DESIGNED COME FROM SOCIAL NETWORKS

• POPULATION CASCADES

SCENARIO: IN A NETWORK THE BELIEFS OR PAYOFFS OF A PERSON CAN BE INFLUENCED BY THE DECISION OF EVERY OTHER PERSON

THE CENTRAL ISSUE IS THAT A SMALL NUMBER OF DECISIONS BY EARLY ADOPTERS CAN TRIGGER LARGE CASCADES

MODELS: THERE ARE TWO BASIC METHODS THROUGH WHICH BEHAVIORS CAN DIFFUSE (SPREAD)

- INFORMATIONAL EFFECTS: BY OBSERVING THE DECISIONS OF THE OTHERS, PEOPLE REVISE THEIR BELIEFS ABOUT THE MERIT OF ONE BEHAVIOR

  FOR INSTANCE: WE MIGHT REVISE OUR BELIEF ~~[illegible]~~ ABOUT THE RELATIVE VALUE OF AN ANDROID PHONE VS AN iPHONE UPON THE ACTIONS OF OUR FRIENDS

- DIRECT EFFECTS: ADOPTING ONE BEHAVIOR CAN HAVE A DIRECT ~~EFFECT~~ ON THE PAYOFFS OF THE OTHER PEOPLE. MORE PEOPLE ADOPT A CLOUD TECHNOLOGY MORE USEFUL THE TECHNOLOGY.

<u>MODEL</u> [INFORMATION CASCADES]

- NETWORKS OF AGENTS
- EACH AGENT CAN PERFORM TWO ACTIONS $\{0,1\}$
- ONE ACTION IS BETTER THAN THE OTHER, BUT WHICH IS THE BEST IS UNCERTAIN
- EACH PERSON CHOOSES ONE OR THE OTHER ACTION IN SEQUENCE
- EACH PERSON INITIALLY BELIEVES THAT EACH ACTION IS EQUALLY LIKELY TO BE BEST AND THEN RECEIVES A NOISY SIGNAL THAT PROVIDES A PIECE OF INFORMATION ABOUT THE RELATIVE QUALITY.
  THE SIGNAL IS CORRECT WITH PROBABILITY $p > 1/2$
- ONCE RECEIVED THE SIGNAL, EACH PERSON UPDATES THE BELIEF

<u>EXAMPLE</u>: SIGNAL "0 IS BETTER THAN 1" WITH PROBABILITY p

$\Rightarrow$ THE AGENT WILL PLAY 0

DIRECT-EFFECT CASCADES

MODEL: • EACH PERSON IS A 0-TYPE OR A 1-TYPE

* 0-TYPE PREFERS ACTION 0 UNLESS ENOUGH OTHER PEOPLE CHOOSE 1 RATHER THAN 0

* 1-TYPE PREFERS ACTION 1 UNLESS ...

- $p^{(0)} > 0$ PROBABILITY OF BEING 0-TYPE

- EACH PERSON $i \in \{1, 2, 3, \dots\}$ MAKES A SINGLE DECISION IN SEQUENCE

- UTILITY PERSON i:

$$\text{0-TYPE} \longrightarrow u_i = \begin{cases} \beta + \gamma \cdot h_0 & a_i = 0 \\ \alpha + \gamma \cdot h_1 & a_i = 1 \end{cases}$$

$\alpha, \beta, \gamma > 0$ and $\beta > \alpha$ (BEING 0-TYPE)

$$\text{1-TYPE} \longrightarrow u_i = \begin{cases} \beta + \gamma h_1 & a_i = 1 \\ \alpha + \gamma h_0 & a_i = 0 \end{cases}$$

THEREFORE 0-TYPE WILL CHOOSE ACTION 1 IF AND ONLY IF $\alpha + \gamma h_1 \geq \beta + \gamma h_0 \Rightarrow$

$$h_1 - h_0 \geq \frac{1}{\gamma}(\beta - \alpha)$$

AND CHOOSE ACTION 0 OTHERWISE

IF THERE IS A TIE, AN ~~AGENT~~ FOLLOWS THE MAJORITY

- $\text{GAP} = \left\lceil \frac{1}{\gamma}(\beta - \alpha) \right\rceil$ WHEN $h_1 - h_0 \geq \text{gap} \Rightarrow$ THE PROCESS locks-in TO ACTION 1

IF $h_0 - h_1 \geq \text{gap} \Rightarrow$ locks-in TO ACTION 0

- IF $p^{(0)} < 1/2$ AND MORE PEOPLE PREFER ACTION 1 TO ACTION 0, A 0-CASCADE CAN OCCUR WITH PROBABILITY $\geq (p^{(0)})^{\text{gap}}$

OBSERVATION: CASCADE CAN OCCUR EASILY AND CAN BE INEFFIC

NETWORK GAMES

MODEL:
- EACH AGENT IS ASSOCIATED WITH A NODE IN A NETWORK, CONNECTED WITH UNDIRECTED EDGES
- BINARY ACTIONS $\{0,1\}$
- THE UTILITY OF AN AGENT IS AFFECTED ONLY BY ITS NEIGHBORHOOD
- THE UTILITY OF AN AGENT DEPENDS ONLY ON THE NUMBER OF 0s AND 1s TAKEN BY THE NEIGHBORS

MODEL: TWO MODELS BUILD OVER THE BASIC ABOVE

- STRATEGIC COMPLEMENTS: AGENTS [redacted] BENEFIT FROM CHOOSING THE SAME ACTION AS THEIR NEIGHBORS

- STRATEGIC SUBSTITUTES: AGENTS HAVE A LOSS FROM CHOOSING THE SAME ACTION AS THEIR NEIGHBORS

MODEL: [GAMES WITH STRATEGIC COMPLEMENTS]

- BINARY ACTIONS $\{0,1\}$
- $u_i(1, a_{-i}) - u_i(0, a_{-i}) \geq u_i(1, a'_{-i}) - u_i(0, a'_{-i})$

WHERE $a_{-i} \geq a'_{-i} \iff a_j \geq a'_j$ for all $j \neq i$

THE RELATIVE BENEFIT OF PLAYING 1 OVER 0 INCREASES AS MORE OTHER AGENT PLAY 1's

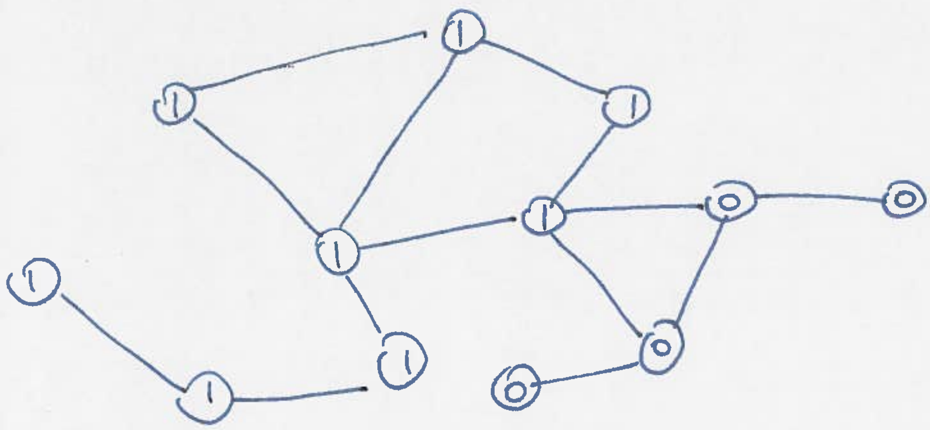
EXAMPLE: [COORDINATION GAME]

	0	1
0	q, q	$0, 0$
1	$0, 0$	$1-q, 1-q$

$q \in (0,1)$

THE UTILITY IS ADDITIVE: FOR EACH AGENT IN ITS NEIGHBORHOOD an agent RECEIVES q IF IT PLAYS 0 WHEN THE OTHER PLAYS 0, AND $1-q$ for 1.

IT CAN BE APPLIED TO A NETWORK:



$q = 1/2$

↓

Nash Equilibrium

MODEL: GIVEN i, GIVEN a_{-i}, LET $n_0(a_{-i})$, $n_1(a_{-i})$ THE NUMBER OF NEIGHBORS PLAYING 0 AND THE NUMBER OF NEIGHBORS PLAYING 1

ACTION 1 IS A BEST RESPONSE IF AND ONLY IF

$$q \, n_0(a_{-i}) \le (1-q) \, n_1(a_{-i})$$

THAT IS

$$\frac{n_1(a_{-i})}{n_0(a_{-i}) + n_1(a_{-i})} \ge q$$

IF $q = \frac{1}{2}$ $\Rightarrow$ MAJORITY GAME

- THERE ARE MANY NASH equilibria INCLUDING ALL-0 AND ALL-1, BUT THERE ARE ADDITIONAL NASH equilibria

MODEL [GAMES WITH STRATEGIC SUBSTITUTES]

- BINARY ACTIONS $\{0,1\}$
- $u_i(1, a_{-i}) - u_i(0, a_{-i}) \leq u_i(1, a'_{-i}) - u_i(0, a'_{-i})$

WHERE $a_{-i} \geq a'_{-i} \iff a_j \geq a'_j$ for all $j \neq i$

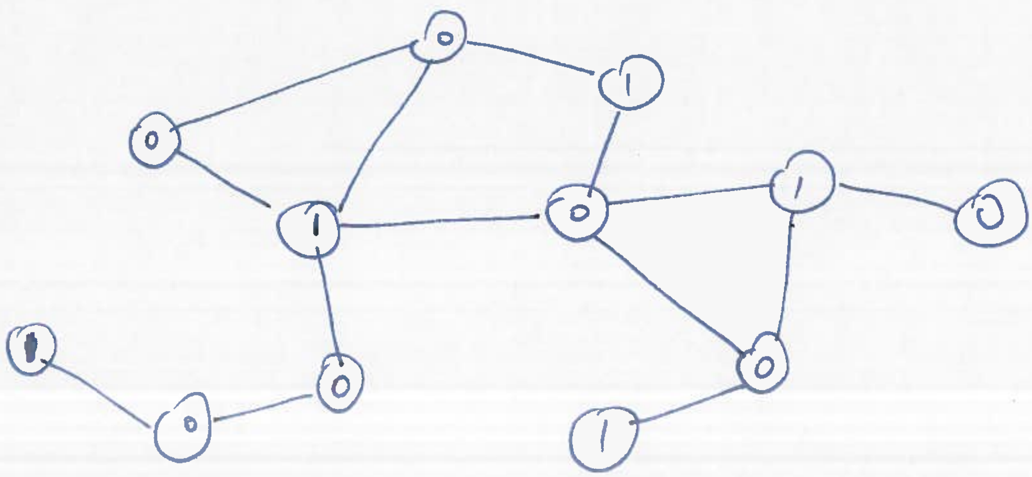
THE RELATIVE BENEFIT OF PLAYING 1 OVER 0 DECREASES AS MORE OTHER AGENTS PLAY 1.

EXAMPLE [LOCAL PUBLIC GOOD]

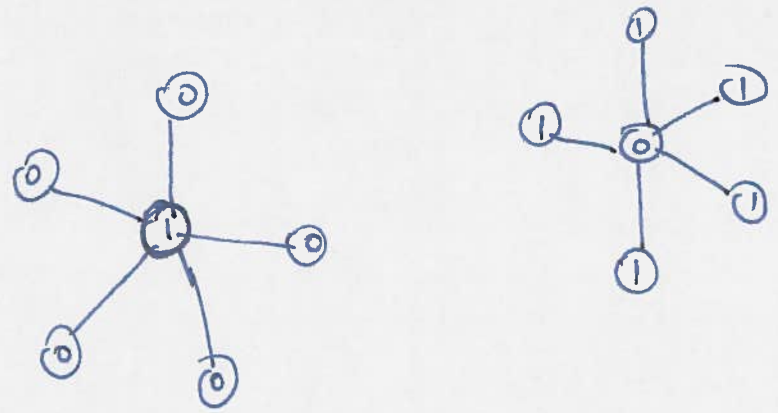
$$u_i(a_i, a_{-i}) = \begin{cases} 1 - c & a_i = 1 \\ 1 & a_i = 0,\ n_i(a_{-i}) > 0 \\ 0 & a_i = 0,\ n_i(a_{-i}) = 0 \end{cases}$$

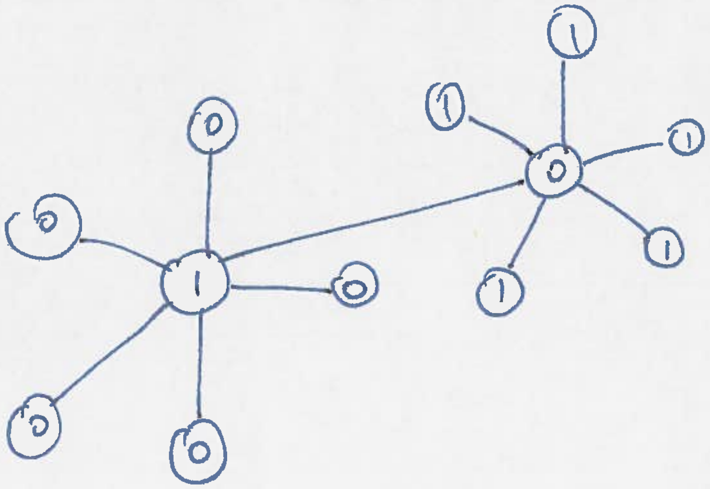
$c \in (0,1)$ EXPRESSES THE COST OF CHOOSING THE ACTION 1

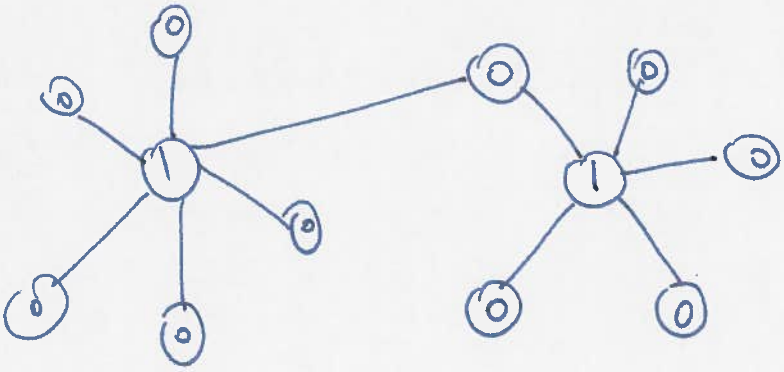
SINCE $c > 0 \Rightarrow$ IN EVERY NEIGHBORHOOD AT LEAST ONE AGENT TAKES ACTION 1, BUT NO MORE THAN ONE AGENT



EXAMPLE:







ADDING DIFFERENT EDGES PRODUCES COMPLETELY DIFFERENT EQUILIBRIA

OBSERVATION: Equilibria with all 0s or all 1s cannot exist

PROPERTY: AT THE NASH Equilibrium, THE ~~AGENTS~~ WHO TAKE 1 FORM A MAXIMAL INDEPENDENT SET

OBSERVATION:

- IN GAMES WITH STRATEGIC COMPLEMENTS, THERE MUST BE SOME THRESHOLD FOR MAKING AN AGENT TO CHANGE ACTION TOWARD THE ACTIONS OF THE OTHERS

- COMPLEMENTARY

CASCADING BEHAVIOR IN NETWORKS

MOTIVATION: THE GOAL NOW IS TO UNDERSTAND WHEN THE EFFECT OF INFLUENCING SOME SMALL SET OF AGENTS TO SWITCH TO ACTION 1 CAN LEAD TO A CASCADE OVER THE NETWORK

MODEL: [HARD-THRESHOLD CASCADE]

- CONNECTED GRAPH (UNDIRECTED)
- ALL THE AGENTS INITIALLY PLAY 0
- QUESTION: WHEN A CASCADE ACTIVATES?

THRESHOLD : $q \cdot d$ (WHERE d IS THE DEGREE OF AN AGENT)

IF AT LEAST A FRACTION q OF NEIGHBORS PLAY 1 $\Rightarrow$ 1 $\Rightarrow$ AT LEAST qd NEIGHBORS

- CONDITIONS NEEDED TO HAVE AN equilibrium IN WHICH SOME AGENTS PLAY 0 AND THE OTHERS 1
- N agents, $S \subsetneq N$ agents playing 1, $T = N / S$ agents playing 0

  * Each $i \in S$ HAS AT LEAST A FRACTION q OF NEIGHBORS IN S

  * Each $i \in T$ HAS AT LEAST A FRACTION $1-q$ OF NEIGHBORS IN T

COHESIVENESS OF S: $coh(S) = \min_{i \in S} \frac{|N_i \cap S|}{|N_i|}$

N_i IS THE SET OF NEIGHBORS OF i

r-COHESIVE WITH $r \in [0,1]$ IF $coh(S) \geq r$

THEOREM: A NETWORK G CAN SUPPORT A Nash equilibrium WITH BOTH 0 and 1 $\Leftrightarrow$ THERE EXISTS A SET $S \subsetneq N$ such THAT S IS q-COHESIVE AND T IS $(1-q)$ COHESIVE

QUESTION: IF WE ACTIVATE AN INITIAL SET S_1 TO PLAY 1, CAN WE CAUSE A CASCADE such that all agents PLAY

MODEL:
- ACTIVATE AN INITIAL SET S_1 to play 1 (e.g., thanks TO INCENTIVES) AND FIX THE BEHAVIOR OF THESE AGENTS
- ALL THE OTHER AGENTS ARE INITIALIZED TO PLAY 0
- AT $t \in \{1, 2, 3, \dots\}$ CONSIDER ALL THE AGENTS AND IF SOME AGENT PLAYING 0 HAS 1 as BEST RESPONSE THEN SWITCH THE AGENT

AGENTS ARE MYOPIC, NOT CONSIDERING THE FUTURE EFFECT

OBSERVATION: THE SUCCESS OF 1-CASCADE DEPENDS ON:
- SEED SET S_1
- FRACTION q required FOR 1 TO BE BEST RESPONSE
- TOPOLOGY OF THE NETWORK

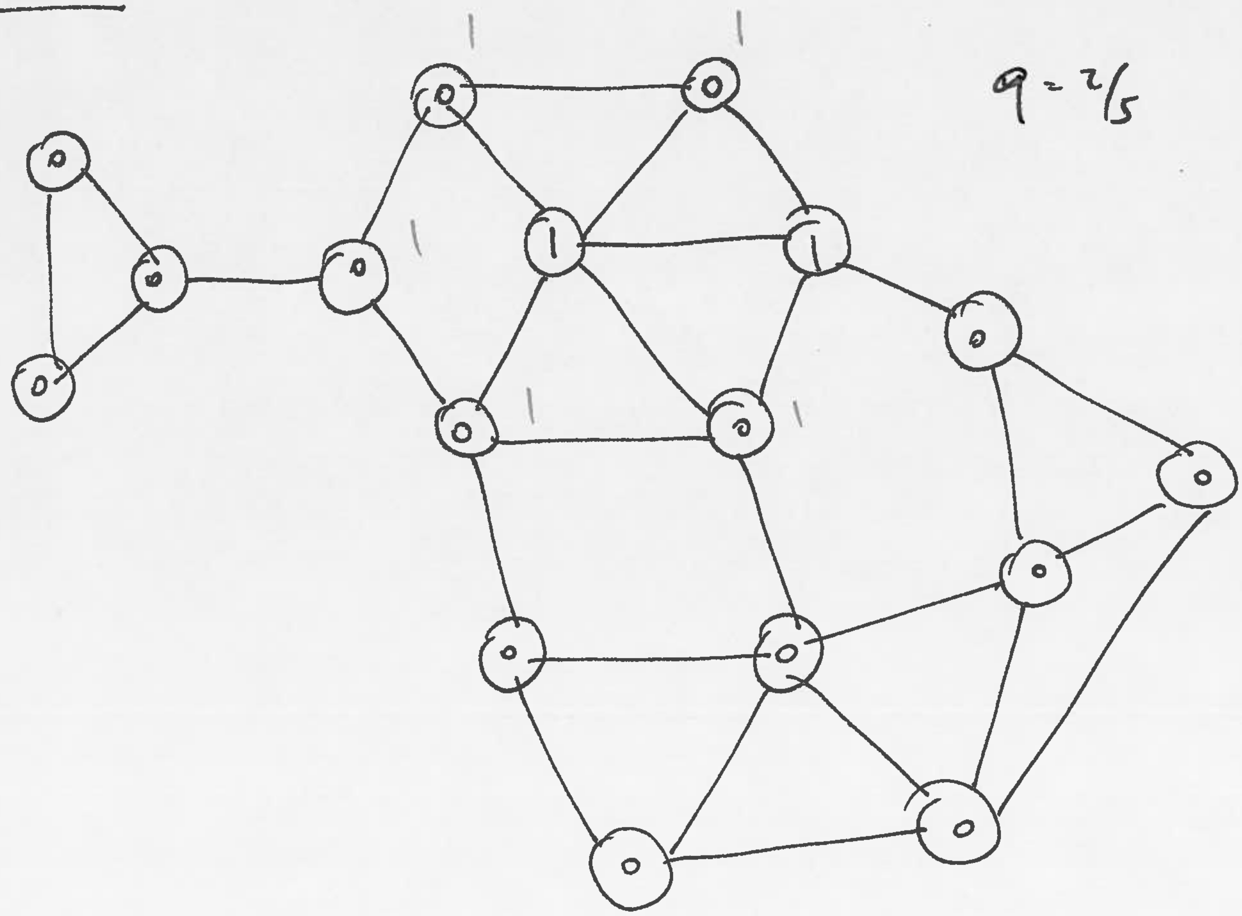
THEOREM: THE SET OF INITIAL ADOPTERS S_1 playing 1 CAN CAUSE a COMPLETE 1-CASCADE AT THRESHOLD q $\Leftrightarrow$ $N \setminus S_1$ CONTAINS NO SUBSET WITH COHESIVENESS $> 1-q$

OBSERVATION: A 1-CASCADE CAN SUCCEED AS LONG AS THERE IS NO SUBSET OF AGENTS WITH INTERNAL COHESIVENESS LARGE ENOUGH TO SUSTAIN 0 AND BLOCK THE PROGRESS OF ACTION 1 EVEN IF ALL THE AGENTS OUTSIDE OF THE SUBSET PLAY 1

OBSERVATION:

- THE INITIAL ADOPTERS S, SHOULD BE SELECTED TO CONTAIN THE MOST COHESIVE PARTS OF THE NETWORK, WITH THE REST OF THE NETWORK BEING LESS COHESIVE
- NETWORKS WITH LESS COHESION ARE EASIER TO TIP
- A SMALLER q IS BETTER FOR THE CASCADE

EXAMPLE:



MODEL [SOFT-THRESHOLD CASCADE]:

- DIRECT GRAPH WITH WEIGHTS $p_{ij} \in [0,1)$
- AN AGENT IS ACTIVE IF IT ADOPTS ACTION 1
- p_{ij} probability THAT i CAN INFLUENCE j TO SWITCH FROM 0 TO 1
- ACTIVATE A SET S, TO PLAY 1, ALL THE OTHER PLAY 0
- IN EACH TIME $t \in \{1, 2, 3 \ldots\}$ ANY ACTIVATED AGENT FOR THE FIRST TIME AT $t-1$ HAS A CHANCE TO ACTIVATE A NEIGHBOR $j \in N_i$ THAT IS NOT ACTIVE WITH PROBABILITY p_{ij}
- THIS CONTINUES UNTIL NO NEW AGENTS ARE ACTIVATED

INFLUENCE MAXIMIZATION

MODEL:

- INITIAL SET S
- $I(S)$ IS THE RANDOM SET OF NODES ACTIVATED BY THE END OF THE PROCESS
- $f(S) = \mathbb{E}[|I(S)|]$ IS THE INFLUENCE FUNCTION DENOTING THE EXPECTED NUMBER OF ACTIVATED NODES
- GOAL:

$$\max_{\substack{S \subseteq N \\ |S| = K}} f(S)$$

THAT IS: FIND THE INITIAL SET S OF SIZE K THAT MAXIMIZES THE EXPECTED NUMBER OF ACTIVATED NODES.

PROPERTY: THE PROBLEM IS NP-HARD

(THE REDUCTION IS FROM SET-COVER)

OBSERVATION: THE PROBLEM CAN BE APROXIMATED SINCE $f(S)$ IS SUBMODULAR. THAT IS:

$$f(S \cup \{i\}) - f(S) \geq f(T \cup \{i\}) - f(T)$$

for all $S \subset T$ AND $i \notin S$

(AS THE SET TO WHICH i IS INTRODUCED GETS LARGER, THE MARGINAL BENEFIT OF i IS WEAKLY DECREASING)

ALGORITHM:

- $S = \emptyset$
- FOR $t = 1 \ldots K$
  - SELECT $i \in N / S$ TO MAXIMIZE $f(S \cup \{i\}) - f($
  - $S \leftarrow S \cup \{i\}$

Property: THE ALGORITHM PROVIDES AN APPROXIMATION OF $\frac{e-1}{e}$ WHEN f IS MONOTONE AND SUBMODULAR

BUYING INFLUENCE

- FORMALLY EACH PERSON $i \in \{1, 2, \dots\}$ MAKES A SINGLE DECISION IN SEQUENCE WITH KNOWLEDGE OF THEIR OWN PRIVATE SIGNAL AND THE SEQUENCE OF OTHER DECISIONS MADE SO FAR

EXAMPLE:

- PERSON 1: IT PLAYS 0 BECAUSE ITS SIGNAL PUSHES FOR 0
- PERSON 2: IF ITS SIGNAL IS 0 $\Rightarrow$ IT PLAYS ZERO GIVEN THAT THE SIGNAL OF PERSON 1 STRENGTHENS THE BELIEF OF PERSON 2

  IF ITS SIGNAL IS 1 $\Rightarrow$ IT PLAYS 1 BECAUSE THERE IS A TIE AND PERSON 2 BREAKS THE TIE IN FAVOR OF ITS BELIEF
- PERSON 3: IT CAN INFER THE SIGNALS OF THE TWO PREVIOUS PEOPLE

  * IF $\langle 0, 0 \rangle \Rightarrow 0$ BECAUSE THERE ARE AT LEAST TWO SIGNALS OF 0 VS AT MOST ONE SIGNAL OF 1

  * IF $\langle 0, 1 \rangle \Rightarrow$ IT DEPENDS ON THE SIGNAL OF PERSON 3. IN PRACTICE THE FIRST TWO SIGNALS CANCEL EACH OTHER
- PERSON 4: IT CAN INFER THE SIGNALS OF THE FIRST 3 PREVIOUS PEOPLE

  * IF $\langle 0, 0, 0 \rangle \Rightarrow 0$ INDEPENDENTLY OF ITS SIGNAL

    $\Rightarrow$ ONCE $h_0 - h_1 \geq 2 \Rightarrow$ CASCADE OF 0

    HERDING

  * IF $\langle 0, 1, 0 \rangle$ OR $\langle 0, 1, 1 \rangle \Rightarrow$ IT DEPENDS ON THE SIGNAL

CASCADE $\Rightarrow$ IN FUTURE THE DECISIONS WILL BE THE SAME INDEPENDENTLY OF THE SIGNALS

PROPERTY: THE PROBABILITY OF A CASCADE GOES TO 1 AS THE NUMBER OF PLAYERS INCREASES

- CONSIDER $3m$ PLAYERS
- WHAT IS THE PROBABILITY THAT THREE SUBSEQUENT PLAYERS RECEIVE THE SAME SIGNAL?

$$\bar{p} = p^3 + (1-p)^3$$

- WHEN 3 SUCCESSIVE PLAYERS RECEIVE THE SAME SIGNALS, A CASCADE OCCURS ($\Rightarrow$, BUT $\not\Leftarrow$)
- WHAT IS THE PROBABILITY THAT NO THREE SUCCESSIVE PLAYERS HAVE THE SAME SIGNAL?

$(1-\bar{p})^m$ THAT GOES TO ZERO AS $m \to \infty$

MORE COMPLICATED SITUATIONS:

- DIFFERENT p
- DIFFERENT PREFERENCES
- MORE ACTIONS
- NETWORKS