LEARNING TO BID IN SEQUENTIAL DUTCH AUCTIONS.

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The Research Topic

What path prices for successive sales of identical objects should follow during a sequence of auctions?

In particular, understanding the formation of prices in a sequence of Dutch (descending price) auctions.
Theoretical literature

Milgrom and Weber (1982) show that in equilibrium if bidders have independently drawn private values successive sales of identical objects should yield, on average, the same price.

As van den Berg et al. (2001) point out if $n$ buyers are faced with the successive sales of $n$ identical objects each of whom wish to buy one, there is a tension between two forces.

- As each round passes one buyer is eliminated from the market suggesting that competition will decline.
- However one object is also removed so the available supply diminishes.

These two tensions should offset each other. If there was a pattern to the sequence of prices, then those participating in the auction at the time when the price was highest would have done better to participate at the lower price period.
Empirical studies: “the declining price anomaly”

The empirical evidence suggests otherwise.

In many markets organised as successive auctions prices have been observed to decline over time and, as a result, this has been called the “declining price paradox”.

In particular, two stylised facts are recurrent in the empirical literature:

- the “declining price paradox”
- and the fact that, at the very end of the day prices may rise.
Why Fish market?

Perishable goods markets are particularly suited to the analysis of price formation since they have a number of “good” properties.

Fish is perishable and therefore there is no possibility for the sellers to postpone the sale of the good (fresh fish cannot be stocked for more than a day and still be characterised as “fresh”), so the total quantity on hand must be traded.

This greatly simplifies the economic analysis since one does not have to analyse the holding of inventories by the seller and can reduce his behaviour to selling to the current best bidder with no intertemporal considerations.
Sequential Dutch auctions.

A standard market model inspired by Fish markets:

- The market is repeated on a daily basis with the same sellers and the same buyer each day. The market participants are all professionals.

- The order of the presentation of the sellers’ goods is chosen randomly every morning and assigned to a specific conveyor belt. The buyers learn the sellers’ order over time and do not know it at the beginning of the day.

- Nobody knows the global offer but the buyers can estimate it (Signal coming from the weather, approximation of stock they see on the market).

- For each auction, the seller sets the minimum (reservation) price, which is known only to the auctioneer.

- Maximum (starting) price is set by the auctioneer.

- The buyers watch the price-clock going down and can bid on one or more of them.

- If no buyers bid until the minimum price is reached, the seller keeps the good. The seller is assumed to sell it on other markets or for processing.
Agent-based Computational Market Model

**Good sold:** one unit of an indivisible, homogenous good per auction

**Market mechanism:** daily sequential Dutch/descending auction

**Market actors:**

- **one auctioneer** playing a **constant strategy** corresponding to the starting price. The rationale is that the auctioneer only decreases starting price for avoiding buyers' strategic reasoning.

- **one auctioneer** who is responsible for the **global offer** assumed to be exogenous, fixed because equal to the number of auction rounds. She is assumed to play a **constant strategy**. The rationale is that the seller submits a reservation price equal to a constant opportunity cost

- **n learning buyers** who constitute the global demand. The rationale is that the buyers exhibit the most variable behavior among market actors.

*We focus on the buyers’ behavior*
The behavioral model

The most common agent-based modeling approach is to exploit a well-established behavioral model which determines certain minimal behavioral requirements for the agents decision making process capable of replicating the empirical evidence and in particular certain “stylized facts”.

Standard approaches/algorithms are adopted for different economic contexts where the level of information available varies or where the repeated nature of the interaction among the same market actors is neglected.
The behavioral model

We propose a behavioral models incorporating not only the well exploited adaptive assumptions, but also more sophisticated learning assumptions to capture salient aspects of the specific context modeled which have been derived also from field study and data analysis.

The idea is to validate the model not only from a so-called predictive or descriptive output validation perspective, but also to ensure that the structural conditions, institutional arrangements, and behavioral dispositions incorporated into the model reflect the salient aspects of the actual system.
Buyers’ behavioral model

We propose a behavioural model with the following features:

- Buyers are daily profit maximizers and considers marginal profits
- Buyers are adaptive according to a reinforcement learning model

We assume that the random order of presentation of the sellers does not allow loyalty between buyers and sellers to develop. Even if some statistical analysis, (Giulioni, G. and Bucciarelli, E. (2007)) found a non negligible presence of loyalty.
Buyers' profit optimising model

Buyer $i$ obtains profits equal to:

$$\Pi^i_d(p^i_k, Q^i) = Q^i \frac{a_i - Q^i}{b_i} - \sum_{k=1}^{K} p^i_k$$

- $Q^i$ is the number of units bought in the wholesale market, which she then sells in the retail market.

- **Final revenue is realised** in the retail market, where the buyer in the auction who now sells, is supposed to face a linear demand:

  $$D^i_{d}^{i,r}(p^i_{d,r}) = Q^i = a_i - b_i p^i_{d,r}$$

- **Costs** corresponding to the daily sum of prices paid in the wholesale market/Dutch auction.
Buyers' profit optimising model

Buyers obtain rewards at each auction equal to marginal profits of buying a further unit of good at the auction and then selling on the retail market:

\[
\pi_i(q^i + 1) = \Pi_{t+1}^n(q^i + 1) - \Pi_t^n(q^i) = \frac{a_i - (2q^i + 1)}{b_i} - p_t^i
\]

We suppose that a buyer exits the current daily auction when the marginal profit at price 0 is negative, i.e.:

\[
Q^{*,i} > \frac{a_i - 1}{2}
\]
Demand models
Demand models
Buyers' learning model

The features of the adaptive learning model we have considered:

• Model with **actions** and **states** (condition-action rule)

• **Reinforcement** learning rule (explore and exploit mechanisms)

• **Counterfactual** reasoning

• **Intertemporal** optimization

we propose an **extension of the q-learning algorithm**
Buyers' learning model

**Action space:** \( a^i \in A^i \)
- A discrete set of prices corresponding to different reservation prices.

**State space:** \( s^i \in S^i \)
- the buyer modifies her reservation price conditional on the amount of units she has already obtained (internal state).
- the buyer modifies her reservation price conditional on the time lasting until the daily market close (external state).
- the total state space is given by the Cartesian product of internal and external state sets.
Action-state representation

An Agent-based Computational Model for Sequential Dutch Auctions.
Action-state representation

An Agent-based Computational Model for Sequential Dutch Auctions.

(Y (Number of units bought))

(X (Auction rounds))

New Day

1,1
1,2
1,3
1,4
1,5
1,6
1,7
1,8

0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8

A
Counterfactual reasoning

As in the original EWA learning model, the buyers is supposed to reinforce not only the most recently played action but all other actions.

- If the buyer lost:
  1. she can easily understand that any other private reservation prices that she would have had below the market price would have yielded an identical profit equal to zero and the same future state.
  2. she also knows what would have happened if her reservation price had been above the market price, that is, she would have obtained the fish at that price.

- If the buyer won:
  1. she can easily understand that had she had any other private reservation prices above her own winning bid price. This would have yielded a similar outcome of the market game (but different profit) and the same future state.
  2. She cannot infer anything for other bids below her winning bid since she cannot observe what other participants would have bid.
We do not consider a belief-based learning model (in belief based models each buyer explicitly models opponents’ behavior). In our case assuming strategic reasoning would not be reasonable because of
• the high number of buyers,
• the fast decision-making process
• the auctioneer’s strategy of speeding up the trade by diminishing occasionally the starting price.
Extended q-Learning

\[ A^{i}_{t+1}(s^{i}_t,a^{i}_t) = (1 - \alpha) \cdot A^{i}_t(s^{i}_t,a^{i}_t) + \alpha \cdot R^{i}_t(s^{i}_t,a^{i}_t,a^{-i}_t) \]

We assume a classical logit probabilistic choice model for exploring the action space in each state:

\[ \pi^{i}_t(s^{i}_t,a^{i}_t) = \frac{e^{|A^{i}_t(s^{i}_t,a^{i}_t)|}}{\sum_{a^{i}_t \in A^{i}_t} e^{|A^{i}_t(s^{i}_t,a^{i}_t)|}} \]
We propose a temporal difference mechanism (Q-learning) for addressing the issue of intertemporality (it is a repeated task!)

Instead of:

$$R^{i,j}_{t} (s^i_t, a^i_t) = \pi_t \quad \text{TD=0}$$

We have:

$$R^i_t (s^i_t, a^i_t, a^{-i}_t) = \pi_t + \gamma A^*_t (s_t) \quad \text{TD=1}$$

$$A^{*,i}_t (s_t) = \max_{a^i \in A^i_t} A^i_t (s^i_{t+1}, a^i)$$
Simulation Parameters

\[ R = 50 \]

\[ D = 1000 \]

\[ F = 100 \]

\[ T = 8; 12 \]
First set of scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Supply</th>
<th>Demand</th>
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<tbody>
<tr>
<td></td>
<td>Auction rounds</td>
<td>Marg. Profits of learning buyers</td>
</tr>
<tr>
<td>L1</td>
<td>8</td>
<td>linear 10, 6</td>
</tr>
<tr>
<td>I1</td>
<td>8</td>
<td>inelastic 10, 10</td>
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<td>I0</td>
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<td>inelastic 10, 10</td>
</tr>
</tbody>
</table>
One buyer

Supply side
- 8 auction rounds => 8 units

Demand side
- 1 buyer x 2 units = 2 units
  - Linear Case: marginal profits for the two units [10 6]
  - Inelastic Case: marginal profits for the two units [10 10]

- plus one random buyer at each auction round:
  - bidding a price randomly drawn from U[0,10]
Action-state attraction matrix

Y (Number of units bought)

X (Auction rounds)
An Agent-based Computational Model for Sequential Dutch Auctions.

Action-state attraction matrix

Y (Number of units bought)

X (Auction rounds)

States

Prices

Linear demand

Actions
Action-state attraction matrix

Y (Number of units bought)

X (Auction rounds)

States

Prices

Linear demand

Actions

(0,1) (0,2) (0,3) (0,4) (0,5) (0,6) (0,7) (0,8)

(1,2) (1,3) (1,4) (1,5) (1,6) (1,7) (1,8)
Action-state attraction matrix

An Agent-based Computational Model for Sequential Dutch Auctions.

Linear demand

States

Prices
Backward Induction

The optimizing buyer $i$ should maximize $E[\Pi^i(\hat{s}, a)]$ the sum of the future expected profits $\pi^i(s, a)$ starting from the current state $\hat{s}$ up to the end of the “episode” i.e. the final absorbing state $\bar{s}$:

$$E[\Pi^i(\hat{s}, a)] = E[\sum_{s=\hat{s}}^{\bar{s}} \pi^i(s, a)].$$

(7)

It is worth noting that, by construction, no profits can be earned in the final absorbing state $\bar{s}$, thus $\pi^i(\bar{s}, a)$ is always equal to 0.
Backward Induction

Now, assume that the buyer is at state \((0,8)\), that is, she is in the last (eight) auction round and is still considering buying the first unit of good, what is the best strategy?

\[
\max_a E[\Pi^i(s_8^0,a)] = \max_a E[\pi^i(s_8^0,a)] = \max_a [Pr_w(s_8^0,a) \cdot \pi^i(s_8^0,a)]
\]  

(8)

We consider \(a\) as any possible action (a bid price) available at state \(s_8^0 = (0,8)\), and \(Pr_w(s_8^0,a)\) is the probability of winning the auction (obtaining the unit of good) given that the buyer bids \(a\) at current state \(s_8^0\).

With respect to state \(s_8^0\), \(Pr_w(s_8^0,a) = \frac{a}{10}\) and \(\pi^i(s_8^0,a) = 10 - a\)

A straightforward calculation shows that the action yielding the maximum of the sum of the expected profits is a bid price equal to 5. In the bottom plot of Figure 4 for state \(s_8^0\) the black dot indicates a bid of 5. The buyers behavior is optimal with respect to such a state, yet she has done no conscious optimisation.
An Agent-based Computational Model for Sequential Dutch Auctions.

Action-state attraction matrix

Y (Number of units bought)

X (Auction rounds)

Linear demand

Prices

States
An Agent-based Computational Model for Sequential Dutch Auctions.

Action-state attraction matrix

Y (Number of units bought)

X (Auction rounds)

Linear demand
**Action-state attraction matrix**

Linear demand

\[
R_t(s^i, a^j, a^{-i}) = \pi_t + \gamma A_t^*(s_t)
\]

TD=1

\[
R_t(s^i, a^j, a^{-i}) = \pi_t
\]

TD=0

---

An Agent-based Computational Model for Sequential Dutch Auctions.
Action-state attraction matrix

Linear demand

Perfectly inelastic demand
Action-state attraction matrix

Perfectly inelastic demand

\( R_t^i(s_t^i, a_t^i, a_t^{-i}) = \pi_t + \gamma A_t^* (s_t) \)

TD=1

Perfectly inelastic demand

\( R_t^{i,j}(s_t^i, a_t^i) = \pi_t \)

TD=0
## Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Supply</th>
<th>Demand</th>
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<tbody>
<tr>
<td></td>
<td>Auction rounds</td>
<td>Type</td>
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<tr>
<td><strong>2L</strong> four homog. Learning buyers</td>
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<td><strong>3L</strong> four homog. Learning buyers</td>
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<td>12</td>
<td>inelastic</td>
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Four homogeneous buyers

Supply side
- 8 auction rounds => 8 units

Demand side
- 4 homog. buyers x 2 units = 8 units
  - Linear Case: marginal profits for the two units [10 6]
  - Inelastic Case: marginal profits for the two units [10 10]

- plus one random buyer at each auction round:
  - bidding a price randomly drawn from $U[0,10]$
Average action-state attraction matrix

**Linear demand**

<table>
<thead>
<tr>
<th>States</th>
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**Perfectly inelastic demand**

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Price dynamics

Auction rounds

linear demand
inelastic demand
Four homogeneous buyers

**Supply side**
- 12 auction rounds => 12 units

**Demand side**
- 4 homog. buyers x 2 units = 8 units
  - **Linear Case**: marginal profits for the two units [10 6]
  - **Inelastic Case**: marginal profits for the two units [10 10]

- plus one random buyer at each auction round:
  - bidding a price randomly drawn from U[0,10]
## Average action-state attraction matrix

### Linear demand

<table>
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### Perfectly inelastic demand

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An Agent-based Computational Model for Sequential Dutch Auctions.
Price dynamics

Auction rounds

Price

linear demand
inelastic demand
Conclusion

We have reproduced a price declining phenomenon by isolating three potential sources:
- diminishing marginal profits (linear retail demand model),
- time pressure (perfectly inelastic retail demand model)
- excess supply.

All these aspects in a more complex and realistic market setting can jointly contribute to amplify such a price formation phenomenon.

The heterogeneity among buyers (in particular in terms of marginal profits) is an aspect which has not been addressed for the sake of intelligibility of simulation outcomes. This aspect can further contribute to "exogenously" impose the declining price effect on market prices.

Statistical investigation of human-subjects experiments and bidding data of market participants on the real wholesale fish markets would be useful to test our theoretical/computational findings.