ALGORITHMIC NON–COOPERATIVE GAME THEORY EXERCISES

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1. Equilibrium computation

Exercise 1.1 (Kuhn Poker). *Consider the Kuhn Poker game:*

- *there are two players* $\{1, 2\}$*;*
- *there are three cards* $\{K, Q, J\}$*;*
- *each player antes 1;*
- *each player is dealt one of the three cards, and the third is put aside unseen (no player can observe the cad of the opponent);*
- *player 1 can* check *or* bet *1;*
  - *if player 1 checks then player 2 can* check *or* bet *1;*
    - *if player 2 checks there is a showdown for the pot of 2;*
    - *if player 2 bets then player 1 can* fold *or* call*;*
      - *if player 1 folds then player 2 takes the pot of 3;*
      - *if player 1 calls there is a showdown for the pot of 4;*
  - *if player 1 bets then player 2 can* fold *or* call*;*
    - *if player 2 folds then player 1 takes the pot of 3;*
    - *if player 2 calls there is a showdown for the pot of 4;*
- *in the showdown, the player with the highest card wins the pot entirely.*

Answer the following questions.

(1) *Provide the extensive–form representation of the game.*
(2) *Provide the sequence–form representation of the game.*
(3) *Provide the size of the normal–form representation of the game.*
(4) *Provide the formulation of the Maxmin strategy finding problem applied to the sequence–form representation for both players.*
(5) *Find a Maxmin strategy for each player.*

Exercise 1.2 (Simplified bargaining game). *Consider the following simplified bargaining game:*

- *there are two players, one buyer* $\mathbf{b}$ *and one seller* $\mathbf{s}$*;*
- *the seller player is of two types:* $\theta_{\mathbf{s}.1}$ *with a probability of 0.33 and* $\theta_{\mathbf{s}.2}$ *with a probability of 0.67;*
- *the first player can* offer *either 0.33 or 0.66 and the action is perfectly observable;*
- *the second player can* accept *the offer* x *of the first player,* counteroffer $x \pm 0.20$*, and this action is perfectly observable;*
  - *if the second player counteroffers, then the first player can* accept *the offer* x *of the second player,* counteroffer $x \pm 0.10$*, and this action is perfectly observable;*
    - *if the first player counteroffers, then the second player can* accept *or* reject*;*
      - *if the second player accepts, the game concludes with an agreement over the last offer* x*;*
      - *if the second player rejects, the game concludes with a disagreement;*
    - *if the first player accepts, the game concludes with an agreement over the last offer* x*;*
  - *if the second player accepts, then the game concludes with an agreement over the last offer* x*;*
- *the utility of all the players from a disagreement is 0, while the utility of buyer from an agreement* (x, t) *where* t *is the time at which the agreement is achieved, is* $(1 - x)(\delta_{\mathbf{b}})^t$*, while the utility of seller* $\mathbf{s}.i$ *is* $x(\delta_{\mathbf{s}.i})^t$*. Assume that* $\delta_{\mathbf{b}} = 0.5$*,* $\delta_{\mathbf{s}.1} = 0.4$*,* $\delta_{\mathbf{s}.2} = 0.9$*. Each action requires a unitary temporal cost.*

Answer the following questions in both cases the first player is the seller (and the second is the buyer) and the first player is the buyer (and the second is he seller).

(1) *Provide the extensive–form representation of the game.*
(2) *Provide the sequence–form representation of the game.*
(3) *Provide the size of the normal–form representation of the game.*
(4) *Provide the formulation of the Nash equilibrium.*

(5) *Find a Nash equilibrium.*

Exercise 1.3 (Simplified patrolling game). *Consider the following simplified patrolling game:*

- *there are two players, one attacker* **a** *and one defender* **d***;*
- *there is a fully connected graph with four vertices labeled* $\{v_1, v_2, v_3, v_4\}$*;*
- *the defender is initially at* v_1 *and spends one time point to move from any vertex to any another vertex;*
- *at time* 1*, the attacker decides which vertex to attack among* $\{v_2, v_3, v_4\}$ *and moves to it, simultaneously the defender decides the next vertex to visit among* $\{v_2, v_3, v_4\}$*;*
- *at time* 2*, the defender covers the vertex which it got and decides the next vertex to visit among all the vertices not visited yet;*
- *at time* 3*, the defender covers he vertex which it got;*
- *if the defender covers the vertex under attack, then the utility of the defender is 1, while the utility of the attacker is 0, else the utility of the defender is* $1 - \pi(v_i)$ *where* v_i *is the attacked target, while the utility of the attacker is* $\pi(v_i)$ *.*

(1) *Provide the extensive–form representation of the game.*
(2) *Provide the sequence–form representation of the game.*
(3) *Provide the size of the normal–form representation of the game.*
(4) *Provide the formulation of the Maxmin strategy finding problem applied to the sequence–form representation for both players.*
(5) *Provide the formulation of the Nash equilibrium.*
(6) *Find a Maxmin strategy for each player.*
(7) *Find a Nash equilibrium.*